A dynamical model with hysteresis for the homogenization of ferromagnetic laminated cores

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***Abstract* — Limitations inherent to conventional hysteresis models (e.g. Preisach and Jiles-Atherton) prevent accurate loss analysis in ferromagnetic cores. The purpose of this paper is to go beyond theses limitations. A vectorial model of hysteresis with a clear-cut relation to the fundamental principles of thermodynamics is first presented. This material model is then used in a magnetodynamic finite element model in order to resolve the exact mesoscale distribution of magnetic fields and eddy currents across individual laminations in a laminated ferromagnetic core. This across-lamination model accounts for both hysteresis and eddy currents. It allows not only an exact material parameter identification from Epstein Frame or Single Sheet Tester field and loss measurements, but it can also serve as a Representative Volume Element in a multiscale analysis.**

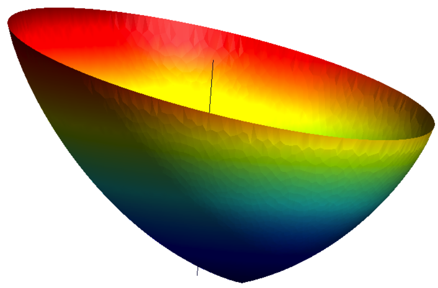

Fig. 1. Representation of the non-smooth convex functional $\Omega(\mathbf{J})$ (4) in the 2D case.

I. INTRODUCTION

Hysteresis effects in electromagnetic simulations are usually accounted for by means of the Preisach and the Jiles-Atherton models [1, 2, 3] or variants thereof. These models suffer however from a number of important limitations. (i) They are empirical, i.e. no physical consideration presides over the choice of the interpolation basis functions they use, except their ability to accurately reproduce the measured data. In consequence, they have poor accuracy when used outside the ranges where measured data is available. (ii) They are scalar. In order to be generalized to 2D or 3D, they must be vectorized, an operation quite artificial and for which a true theoretical basis is lacking. (iii) They are quasistatic. They can represent irreversible hysteresis loops but ignore the effect of frequency on the material's response. (iv) They ignore the fact that magnetic hysteresis is always intertwinned with eddy currents, yielding the practical notion of iron losses, which is the sum of hysteresis (quasistatic) and eddy currents (dynamic) losses. This intertwining not only makes it awkward to identify the parameters of a hysteresis model (like Preisach or Jiles-Atherton), but it also makes its coupling with FE to miss an important part of the phenomenology.

The purpose of this paper is to go beyond these limitations, which prevent accurate loss analysis in ferromagnetic cores. A vectorial model of hysteresis with a clear-cut relation to the fundamental principles of thermodynamics is first presented (Section II.). This model is then used in a magnetodynamic finite element model in order to resolve the exact mesoscale distribution of magnetic fields and eddy currents across individual laminations in a laminated ferromagnetic core. It is shown that this across-lamination model allows an exact material parameter identification from Epstein Frame or Single Sheet Tester measurements (Section IIĨ) at any frequency, Moreover, it can also serve as a Representative Volume Element in a multiscale analysis.

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II. HYSTERESIS MODEL

The proposed model builds on the thermodynamic representation of hysteresis proposed in [4, 5], and which is given a variational formulation, inspired from the kinematic hardening theory of plasticity, in [6, 7]. The energy density u is a function of $\mathbf{J}$ with

$$u = u(\mathbf{J}), \quad \dot{u} = \mathbf{h}_r \cdot \dot{\mathbf{J}}, \quad \text{with} \quad \mathbf{h}_r := \partial_{\mathbf{J}} u. \tag{1}$$

It is used to form the potential $g(\mathbf{h}, \mathbf{J}) := u(\mathbf{J}) - \mathbf{h} \cdot \mathbf{J}$. The dissipation function d describes magnetic hysteresis as a magnetic analogous of a dry friction force can be approximated by the pseudo-potential

$$D(\mathbf{J}, \mathbf{J}_p) := \chi |\mathbf{J} - \mathbf{J}_p| \tag{2}$$

with

$$\partial_{\mathbf{J}} D = \chi \frac{\mathbf{J} - \mathbf{J}_p}{|\mathbf{J} - \mathbf{J}_p|} \approx \chi \frac{\dot{\mathbf{J}}}{|\dot{\mathbf{J}}|} = \mathbf{h}_i, \tag{3}$$

where $\mathbf{J}_p$ is the value of $\mathbf{J}$ at the previous time step. The updated value of $\mathbf{J}$ at each time step follows now from the minimization of the functional

$$\Omega(\mathbf{h}, \mathbf{J}, \mathbf{J}_p) = g(\mathbf{h}, \mathbf{J}) + D(\mathbf{J}, \mathbf{J}_p), \tag{4}$$

which is convex but non smooth with a singular point in $\mathbf{J} = \mathbf{J}_p$ (Fig. 1).

The minimization of (4) is equivalent to solving the vector differential equation

$$\mathbf{h} - \partial_{\mathbf{J}} u - \chi \frac{\dot{\mathbf{J}}}{|\dot{\mathbf{J}}|} = 0, \tag{5}$$

which can be given the graphical representation depicted in Fig. 2. The vector $\mathbf{h}_r$ is linked to the magnetization of the material $\mathbf{J}$ by (1), and the tip of the applied field $\mathbf{h}$ must remain located inside a sphere (a circle in the 2D case) of radius $|\mathbf{h}_i| = \chi$ centered at the tip of $\mathbf{h}_r$.

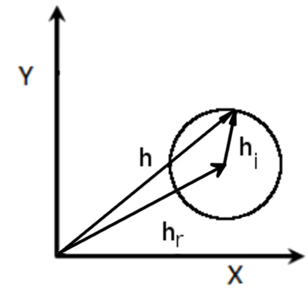

Fig. 2. Graphical representation of the vector equation (5).

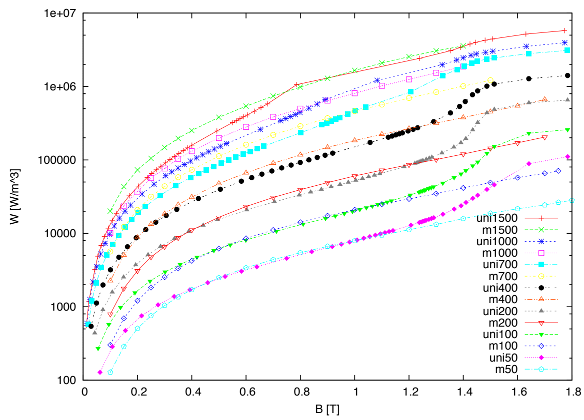

Fig. 4. Matching between measured and computed losses at various frequencies.

The variational formulation (4) provides a robust and coherent framework to efficiently handle the strong nonlinearity of the problem within a finite element scheme. It is however computationnally demanding as a multidimensional optimization must be carried out at each point and each time step. The differential formulation (5), on the other hand, is more intuitive. It accepts a number of approximations that can be solved explicitly, with a significative gain in computation time. A comparison in terms of accuracy and computation time will be done in the full paper.

III. MATERIAL PARAMETER IDENTIFICATION

Both versions of the hysteresis model have been implemented in the 1D magnetodynamic finite element model of an individual lamination in a ferromagnetic core. The finite element formulation will be detailed in the full paper, but one can already note that the global quantities associated with this model (the applied surface magnetic field H_{surf} and the average induction field across the lamination $B_{average}$) correspond exactly with the measured quantities in an Epstein Frame or a Single Sheet Tester.

As the model resolves the mesoscale distribution of magnetic fields and eddy currents, an exact material parameter identification from Epstein Frame (or Single Sheet Tester) field and loss measurements is made possible. The accuracy of this identification depends now on the representation of the statistical distribution of the pinning point strengths in the material. The characteristics of this distribution vary largely across the different types of (soft and hard) ferromagnetic materials. It is accounted in the model by dividing the material magnetization $\mathbf{J}$ into N parts $\mathbf{J} = \sum_{k=1}^{N} \mathbf{J}^k$ and defining for each part $\mathbf{J}^k$ a pinning force χ^k. The functional Ω (4) becomes a sum of independent Ω^k functionals

$$\Omega = \sum_{k=1}^{N} \left(u^k(\mathbf{J}^k) - \mathbf{h} \cdot \mathbf{J}^k + \chi^k |\mathbf{J}^k - \mathbf{J}_p^k| \right) \tag{6}$$

that can be minimized separately. Knowing $\mathbf{h}$ at the current time step, and $\mathbf{J}_p^k$ at the previous time step in each cell, the minimization of Ω^k delivers the updated value of $\mathbf{J}^k$ at the current time step. Fig. 3 shows the graphical representation of Fig. 2 with $N = 10$. Fig. 4 shows that a good match is obtained for iron losses over the whole measured frequency range. The discrepancy at higher fields is due to the fact that a sinusoidal H field was imposed in the model, whereas Epstein measurements are made with a sinusoidal B field.

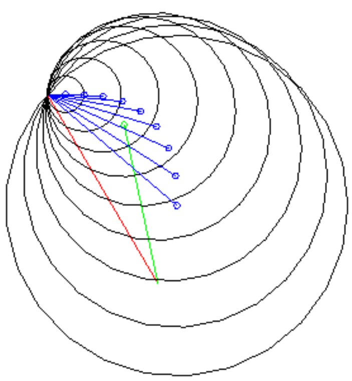

Fig. 3. Graphical representation of the differential model with $N = 10$.

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